Sliding Mode Observers

Theory and Practice

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Outline

1. Conventional Sliding Mode Observers
2. A Simple Sliding Mode Observer
3. Higher Order Sliding Mode Observers
4. Strong Observability - Invariant Zeros - Relative Degree
   - Relation of concepts for SUISO Systems
   - HOSM Observers for LTI systems with unknown inputs
   - Functional HOSM Observers for linear systems
   - Nonlinear case
   - Unknown input identification
5. Output-based stabilization of disturbed systems
   - Estimation of the disturbance.
   - Output-feedback finite-time stabilization of disturbed LTI systems.
6. Fault detection
   - Fault detection using multi-model approach

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Conventional Sliding Mode Observers

Observer Purpose: To estimate the unmeasurable states of a system based only on:

- the measured outputs and inputs;
- mathematical model of the system, driven by the input of the system together with a signal representing the difference between the measured system and observer outputs
- First Observer: Luenberger
- Drawbacks of Luenberguer Observer in the presence of uncertainties
  (a) Unable to force the output estimation error to zero
  (b) The observer states do not converge to the system states
- Solution: sliding mode observer if the uncertainties are bounded.
- Advantages:
  (a) Force the output estimation error to converge to zero in *finite time*
  (b) Observer states converge asymptotically to the system states
  (c) Disturbances can be reconstructed

Observer of Utkin (reduced order SM observer)

- Consider a nominal linear system

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$
$$y(t) = Cx(t) \tag{2}$$

- Assume C has full row rank
- Necessary and sufficient condition: (A, C) is observable
- Observability condition will be assumed to hold.

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- Coordinate transformation $x \mapsto T_c x$

$$T_c = \begin{bmatrix} C^{\perp} \\ C \end{bmatrix} \tag{3}$$

where $N_c \in \mathbb{R}^{n \times (n-p)}$ spans the null-space of C.

- By construction $\det(T_c) \neq 0$
- Applying the change of coordinates

$$T_c A T_c^{-1} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad T_c B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \quad C T_c^{-1} = \begin{bmatrix} 0 & I_p \end{bmatrix} \tag{4}$$

where $A_{11} \in \mathbb{R}^{(n-p)\times(n-p)}$ and $B_1 \in \mathbb{R}^{(n-p)\times q}$.

- Assume the system (1) and (2) is already in the form of (4)
- Utkin proposed observer:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + G_n\nu \tag{5}$$
$$\hat{y}(t) = C\hat{x}(t) \tag{6}$$

  where $(\hat{x}, \hat{y})$ are the estimates of (x, y) and ν is a discontinuous injection term.
- Define $e(t) := \hat{x}(t) - x(t)$ and $e_y(t) := \hat{y}(t) - y(t)$

- Term ν is defined component-wise as

$$\nu_i = \rho \text{sign}(e_{y,i}), \quad i = 1, 2, ..., p \tag{7}$$

where ρ is a positive scalar and $e_{y,i}$ represents the ith component of e_y. (It could be $\nu = \rho e/||e||$).

- ν is designed to be discontinuous with respect to the sliding surface $\mathcal{S} = \{e : Ce = 0\}$ pto force the trajectories of $e(t)$ onto $\mathcal{S}$ in finite time.
- Gain G_n

$$G_n = \begin{bmatrix} L \\ -I_p \end{bmatrix} \tag{8}$$

where $L \in \mathbb{R}^{(n-p)\times p}$ represents the design freedom.

- Error system $\dot{e}(t) = Ae(t) + G_n\nu$
- Partitioning $e = \mathrm{col}(e_1, e_y)$, with $e_1 \in \mathbb{R}^{n-p}$

$$\dot{e}_1(t) = A_{11}e_1(t) + A_{12}e_y(t) + L\nu,\ \dot{e}_y(t) = A_{21}e_1(t) + A_{22}e_y(t) - \nu$$

- Component-wise $\dot{e}_{y,i}(t) = A_{21,i}e_1(t) + A_{22,i}e_y(t) - \rho\ \mathrm{sign}(e_{y,i})$ where $A_{21,i}$ and $A_{22,i}$ represent the ith rows of A_{21} and A_{22}
- Equivalent injection $\nu_{eq} = A_{21}e_1(t)$
- Sliding dynamics $\dot{e}_1(t) = (A_{11} + LA_{21})e_1(t) \rightarrow$ Reduced Order Luenberger Observer

LTI systems with unknown inputs

The system

Consider

$$\Sigma : \begin{cases} \dot{x} = Ax + Bu + Dw, \quad x(0) = x_0, \\ y = Cx, \end{cases} \tag{9}$$

where

- $x(t) \in \mathbb{R}^n$ is the state, $w(t) \in \mathbb{R}^m$ is the unknown input;
- $u(t) \in \mathbb{R}^q$ is the control, $y(t) \in \mathbb{R}^p$ is the measured output.

Strong Observability:

The system is strongly observable if for any $x(0)$ and $w(t)$ it follows from $y(t) \equiv 0 \; \forall \, t \geq 0$ that $x(t) \equiv 0$ [Hautus: 83].

Strong Detectability:

The system is strongly detectable if for any $x(0)$ and $w(t)$ it follows from $y(t) \equiv 0 \; \forall \, t \geq 0$ that $x(t) \to 0$ as $t \to 0$ [Hautus: 83].

LTI systems with unknown inputs

Invariant zeros

The Rosenbrock of $(A,\ C,\ D)$:

$$R(s) = \begin{bmatrix} sI - A & -D \\ C & 0 \end{bmatrix}.$$

The values $s_0 \in \mathbb{C}$ such that $rank\, R(s_0) < n + m$ are called invariant zeros of $(A,\ C,\ D)$.

LTI systems with unknown inputs

State reconstruction without differentiation[Hautus: 1983]

- The system does not have invariant zeroes.
- All the matrices are known i.e.,A, B, C, D.
- C and D are full rank matrices.
- If $rank(C) = p$ and $rank(D) = m$, then $p \geq m$.
- $rank(CD) = m$ Relative degree 1 condition.

Walcot-Zak Observes: Canonical form

$$\begin{pmatrix} dy^{\perp}/dt \\ dy_1/dt \\ dy_2/dt \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix} \begin{pmatrix} y^{\perp} \\ y_1 \\ y_2 \end{pmatrix} + Bu + \begin{pmatrix} 0 \\ 0 \\ w(t) \end{pmatrix}$$

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Mechanical system

Let us consider the following mechanical system:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + P(\dot{q}) + G(q) + \Delta(t,q,\dot{q}) = \tau$$

State space form $x_1 = q$, $x_2 = \dot{q}$, $u = \tau$

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= f(t,x_1,x_2,u) + w(t,x_1,x_2); \qquad y = x_1 \end{aligned}$$

Relative degree condition(linearized case)

$$C = [1 \quad 0], \qquad D = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad CD = 0.$$

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Remark:

When the relative degree of $w(t)$ w.r.t. $y(t)$ is higher than one, i.e. $rank(CD) < m$, output differentiations are necessary.

A simple observer for Mechanical Systems

Formulation of the problem:

Estimate the velocity using the position, under the hypothesis of bounded acceleration.

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = f(x_1, x_2, t) + w, \quad y = x_1.$$

A simple observer [Davila et.al. 05].

The observer

$$\begin{aligned} \dot{\hat{x}}_1 &= -1{,}5\sqrt{L}|y - \hat{x}_1|^{\frac{1}{2}} \operatorname{sign}(y - \hat{x}_1) + \hat{x}_2, \\ \dot{\hat{x}}_2 &= f(x_1, \hat{x}_2, t) - 1{,}1L \operatorname{sign}(y - \hat{x}_1), \end{aligned}$$

$$|f(x_1, \hat{x}_2, t) - f(x_1, x_2, t) + w| < L$$

- finite-time estimation of x_2, i.e., $\hat{x}_2(t) = x_2(t), \forall t \geq T$;
- the best precision in the sense of [Kolmogorov:62].

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Single Unknown Input- Single Output Case

Strong Observability - Invariant Zeros Relation

Strong observability requires that for any input w, the equality $y \equiv 0$ implies $x \equiv 0$. The existence of invariant zeros s_0 implies the existence of inputs $w(s_0)$ such that $y \equiv 0$ for $x \neq 0$

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Absence of invariant zeros is sufficient and necessary condition for strong observability(Haustus,1983)

Single Unknown Input- Single Output Case

Strong Observability - Relative Degree Relation

Taking the first $n-1$ derivatives of the output

$$\begin{aligned} y &= Cx \\ \dot{y} &= C\dot{x} = CAx(t) + CDw \\ &\vdots \\ y^{(n-1)} &= CA^{n-1}x + CA^{n-2}Dw + \ldots + CDw^{(n-2)} \end{aligned}$$

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Relative degree n is required

to obtain a set of n equations independent on w:

$$\begin{bmatrix} CD \\ CAD \\ \vdots \\ CA^{n-2}D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Single Unknown Input- Single Output Case

Invariant zeros - Relative Degree Relation

Rosenbrock matrix for the tuple (A, C, D):

$$R(s) = \begin{bmatrix} sI - A & -D \\ C & 0 \end{bmatrix}.$$

Determinant of the Rosenbrock matrix
$\det(R) = (s^{n-1} + a_n s^{n-2} + ... + a_2)CD +$
$(s^{n-2} + a_n s^{n-3} + ... + a_3)CAD + ... + (s + a_n)CA^{n-2}D + CA^{n-1}D$

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Relative degree n is necessary:

The determinant does not dependent on s iff:

$$\begin{bmatrix} CD \\ \vdots \\ CA^{n-2}D \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$$

Methodology

The Unknown Input Observer (UIO) design problem for strongly observable systems is reduced to evaluate in real-time derivatives of the output.

The k-th order HOSM differentiator for y_j

$$
\begin{aligned}
\dot{z}_0 &= \nu_0 = -\lambda_k L^{\frac{1}{k+1}} |z_0 - y_j|^{\frac{k}{k+1}} \operatorname{sign}(z_0 - y_j) + z_1, \\
\dot{z}_1 &= \nu_1 = -\lambda_{k-1} L^{\frac{1}{k}} |z_1 - \nu_0|^{\frac{k-1}{k}} \operatorname{sign}(z_1 - \nu_0) + z_2, \\
&\vdots \\
\dot{z}_{k-1} &= \nu_{k-1} = -\lambda_1 L^{\frac{1}{2}} |z_{k-1} - \nu_{k-2}|^{\frac{1}{2}} \operatorname{sign}(z_{k-1} - \nu_{k-2}) + z_k, \\
\dot{z}_k &= -\lambda_0 L \operatorname{sign}(z_k - \nu_{k-1}),
\end{aligned}
\tag{10}
$$

$\lambda_0 = 1{,}1$, $\lambda_1 = 1{,}5$, $\lambda_2 = 2$, $\lambda_3 = 3$, $\lambda_4 = 5$, $\lambda_5 = 8$.

Convergence of the HOSM differentiator [Levant:03].

If the gain L satisfies $L > |y_j^{(k+1)}(t)|$ for all t, then $z_i = y_j^{(i)}$ after finite-time.

Uniform Robust Exact First-Order Differentiator [Cruz et. al. 11].

$$\dot{z}_0 = -k_1\phi_1(z_0 - y_j) + z_1, \quad \dot{z}_1 = -k_2\phi_2(z_0 - y_j),$$

where

$$\phi_1(\sigma_0) := \lceil\sigma_0\rfloor^{1/2} + \lceil\sigma_0\rfloor^{3/2},$$
$$\phi_2(\sigma_0) := 0{,}5\,\mathrm{sign}(\sigma_0) + 2\sigma_0 + 1{,}5\lceil\sigma_0\rfloor^2.$$

and $\lceil v\rfloor^p := |v|^p\,\mathrm{sign}(v)$.

Remarks.

- the differentiator is uniform with respect to the initial differentiation error;
- this means that the resulting observer converges in prescribed time;
- useful for systems with strictly positive dwell-time;
- arbitrary order uniform exact differentiator [Angulo et al.: Automatica].

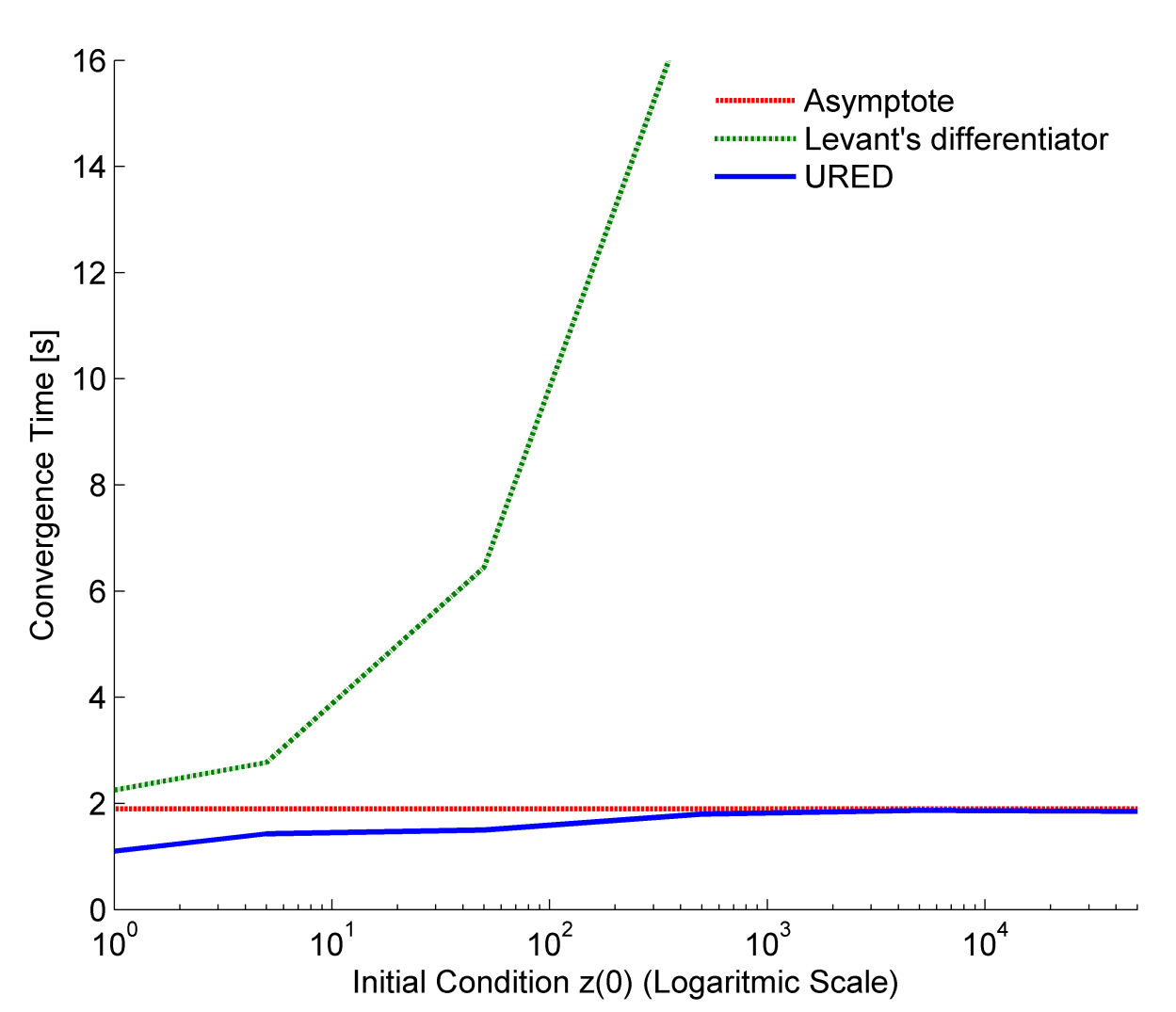

Figura: Convergence time of the Uniform Robust Exact Differentiator (URED).

Arbitrary Order Uniform HOSM differentiator [Angulo et.al: 13, AUtomatica].

Given a signal $y_j(t)$ to be differentiated $(n-1)$-times, the differentiator

$$\begin{aligned} \dot{\hat{x}}_i &= -\lambda_i \lceil y_j - \hat{x}_1 \rfloor^{\frac{n-i}{n}} - k_i \lceil y_j - \hat{x}_1 \rfloor^{\frac{n+\alpha i}{n}} + \hat{x}_{i+1}, \quad i = 1, \ldots, n-1, \\ \dot{\hat{x}}_n &= -\lambda_n \operatorname{sign}(y_j - \hat{x}_1) - k_n \lceil y_j - \hat{x}_1 \rfloor^{1+\alpha}, \end{aligned} \tag{11}$$

with $\{\lambda_i\}_{i=1}^n$ chosen as Levant's, $\alpha > 0$ small enough and $\{k_i\}_{i=1}^n$ such that the polynomial

$$P(s) = k_n s^{n-1} + k_{n-1} s^{n-2} + \cdots + k_2 s + k_1, \quad \text{is stable}$$

provides:

- uniform finite-time estimation, i.e., $\exists T$ independent of $|\hat{x}_i(0) + -y_j^{i-1}(0)|$, $i = 1, \ldots, n$, such that

$$\hat{x}_i(t) = y_j^{(i-1)}(t), \quad \forall t \geq T, \quad i = 1, \ldots, n;$$

- the best precision under measurement noise [Kolmogorov:62].

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LTI systems with unknown inputs

The system

Consider

$$\Sigma : \begin{cases} \dot{x} = Ax + Bu + Dw, \quad x(0) = x_0, \\ y = Cx, \end{cases} \tag{12}$$

where

- $x(t) \in \mathbb{R}^n$ is the state, $w(t) \in \mathbb{R}^m$ is the unknown input;
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Relative degree

The integer r such that

$$\begin{aligned} c_i A^j D &= 0, \; j = 0, ..., r_i - 2, \quad c_i A^{r_i - 1} D \neq 0, \\ r_i &\leq n - 1 \end{aligned}$$

where c_i is the $i$thm row of C.

Observers for LTI systems with unknown inputs

Cascaded HOSM observer [Fridman et al. 2007]

Observer for the strongly observable case and $\sum_{i=1}^{m} r_i = n$

$$\begin{aligned} \dot{z} &= Az + Dw + L(y - Cz), \\ \dot{v} &= W(y - Cz,\, v), \\ \hat{x} &= z + Kv \end{aligned} \tag{13}$$

- $L \in \mathbb{R}^n$ is the correction term chosen such that $A - LC$ is Hurwitz;
- for i=1, ..., m:

$$K^{-1} = \begin{bmatrix} P_1 \\ \vdots \\ P_m \end{bmatrix}, \quad P_i = \begin{bmatrix} c_i \\ \vdots \\ c_i(A - LC)^{n-1} \end{bmatrix};$$

.

- $W(\cdot)$ is a nonlinear HOSM term.

Observers for LTI systems with unknown inputs

Cascaded HOSM observer [Fridman et al. 2007]

Nonlinear HOSM term $W^T = [\ v_1 \quad v_2 \quad \dots \quad v_n\ ]$

$$
\begin{aligned}
\dot{v}_1 &= w_1 = -\alpha_n N^{1/n} |v_1 - (y - Cz)|^{(n-1)/n} sign(v_1 - (y - Cz)) + v_2 \\
\dot{v}_2 &= w_2 = -\alpha_{(n-1)} N^{1/(n-1)} |v_2 - w_1|^{(n-2)/(n-1)} sign(v_2 - w_1) + v_3 \\
&\vdots \\
\dot{v}_{n-1} &= w_{n-1} = -\alpha_2 N^{1/2} |v_{n-1} - w_{n-2}|^{1/2} sign(v_{n-1} - w_{n-2}) + v_n \\
\dot{v}_n &= -\alpha_1 N sign(v_n - w_{n-1})
\end{aligned}
\tag{14}
$$

where $N > |C(A - LC)^{n-1} Dw(t)|$.

Observers for LTI systems with unknown inputs

Canonical form of the estimation error $x - z$

The form is composed for Brunovsky blocks.

$$
\begin{bmatrix}
\dot{\xi}_{1_1} \\ \dot{\xi}_{1_2} \\ \vdots \\ \dot{\xi}_{1_{r_1-2}} \\ \dot{\xi}_{1_{r_1-1}} \\ \vdots \\ \vdots \\ \dot{\xi}_{m_1} \\ \dot{\xi}_{m_2} \\ \vdots \\ \dot{\xi}_{m_{r_m-2}} \\ \dot{\xi}_{m_{r_m-1}}
\end{bmatrix}
=
\begin{bmatrix}
0 & 1 & \cdots & 0 & 0 & \cdots & \cdots & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & \cdots & \cdots & 0 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & & \ddots & & \cdots & \cdots & & \vdots & & \vdots & \\
0 & 0 & \cdots & 0 & 1 & \cdots & \cdots & 0 & 0 & \cdots & 0 & 0 \\
* & * & \cdots & * & * & \cdots & \cdots & 0 & 0 & \cdots & 0 & 0 \\
 & \vdots & & & \vdots & \ddots & & & \vdots & & \vdots & \\
 & \vdots & & & \vdots & & \ddots & & \vdots & & \vdots & \\
0 & 0 & \cdots & 0 & 0 & \cdots & \cdots & 0 & 1 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & \cdots & \cdots & 0 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & & \vdots & & \cdots & \cdots & & \vdots & & \ddots & \\
0 & 0 & \cdots & 0 & 0 & \cdots & \cdots & 0 & 0 & \cdots & 0 & 1 \\
* & * & \cdots & * & * & \cdots & \cdots & * & * & \cdots & * & *
\end{bmatrix}
\begin{bmatrix}
\xi_{1_1} \\ \xi_{1_2} \\ \vdots \\ \xi_{1_{r_1-2}} \\ \xi_{1_{r_1-1}} \\ \vdots \\ \vdots \\ \xi_{m_1} \\ \xi_{m_2} \\ \vdots \\ \xi_{m_{r_m-2}} \\ \xi_{m_{r_m-1}}
\end{bmatrix}
+
\begin{bmatrix}
0 & \cdots & 0 \\
0 & \cdots & 0 \\
\vdots & & \vdots \\
0 & \cdots & 0 \\
* & \cdots & * \\
 & \vdots & \\
 & \vdots & \\
0 & \cdots & 0 \\
0 & \cdots & 0 \\
\vdots & & \vdots \\
0 & \cdots & 0 \\
* & \cdots & *
\end{bmatrix}
\begin{bmatrix}
w_1 \\ w_2 \\ \vdots \\ w_{m-1} \\ w_m
\end{bmatrix}
$$

Observers for LTI systems with unknown inputs

HOSM Observer [Fridman et al. 2007]

Advantages:

1. Finite-time theretically exact observation of the system states
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Question

What can we do when the system is strongly observable but $\sum_{i=1}^{m} r_i < n$?

Weakly unobservable subspace

Invariant zeros

The Rosenbrock of $(A,\ C,\ D)$:

$$R(s) = \begin{bmatrix} sI - A & -D \\ C & 0 \end{bmatrix}.$$

The values $s_0 \in \mathbb{C}$ such that $rank\, R(s_0) < n + m$ are called invariant zeros of $(A,\ C,\ D)$.

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The weakly unobservable subspace $\mathcal{V}^*$

A state $x_0 \in \mathcal{X}$ is called weakly unobservable, if there exist an input w such that the corresponding output $y_w(t, x_0) = 0$ for all $t \geq 0$. The set of all the weakly unobservable points is denoted by $\mathcal{V}^*$ and it is called the weakly unobservable subspace of the system.

Molinari Decoupling Algorithm [Molinari: 1976]

The Molinari's algorithm

The algorithm is given as follows:

- Step 0: Let $i = 0$ and $M_0 = 0$ (a null dimension matrix).

Molinari Decoupling Algorithm [Molinari: 1976]

The Molinari's algorithm

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- Step 0: Let $i = 0$ and $M_0 = 0$ (a null dimension matrix).
- Step 1: Compute $\Gamma_i = \begin{bmatrix} M_i D & M_i A \\ 0 & C \end{bmatrix}$, Find T_i such that Γ_i is reduced to
$$T_i \Gamma_i = \begin{bmatrix} G_{i+1} & H_{i+1} \\ 0 & M_{i+1} \end{bmatrix},$$
where G_{i+1} has full row rank.

Molinari Decoupling Algorithm [Molinari: 1976]

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$$T_i \Gamma_i = \begin{bmatrix} G_{i+1} & H_{i+1} \\ 0 & M_{i+1} \end{bmatrix},$$
where G_{i+1} has full row rank.
- Step 2: Let $i = i + 1$ and back to Step 1.

Remark

The algorithm ends when $rank\ M_i = rank\ M_{i+1}$. The matrix M_i, for which the equality was satisfied, is denoted by M_n.

Molinari Decoupling Algorithm [Molinari: 1976]

Theorem

$$ker\ M_n = \mathcal{V}^*$$

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Importance of the Molinari's algorithm

The algorithm gives an explicit algebraic relation between the output, and its derivatives, and the state.

$$v(y, \dot{y}, ..., y^{(n)}) = M_n x \tag{15}$$

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Importance of the Molinari's algorithm

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$$v(y,\ \dot{y},\ ...,\ y^{(n)}) = M_n x \tag{15}$$

Relations for strong observability

i) The system is strongly observable;

ii) the triplet (A, C, D) does not have invariant zeros;

iii) $\mathcal{V}^*$ contains only the zero vector, i.e. $\mathcal{V}^* = \{0\}$.

Observers for LTI strongly observable systems with unknown inputs

Canonical form

Let $\sum_{i=1} mr_i = r_p < n$. The canonical form is composed by Brunovsky blocks and a w dependent block.

$$
\begin{bmatrix}
\dot{\xi}_{1_1} \\ \dot{\xi}_{1_2} \\ \vdots \\ \dot{\xi}_{1_{r_1-2}} \\ \dot{\xi}_{1_{r_1-1}} \\ \vdots \\ \vdots \\ \dot{\xi}_{p_1} \\ \dot{\xi}_{p_2} \\ \vdots \\ \dot{\xi}_{p_{r_p-2}} \\ \dot{\xi}_{p_{r_p-1}} \\ \dot{\xi}_{r_p+1} \\ \vdots \\ \dot{\xi}_n
\end{bmatrix}
=
\begin{bmatrix}
0 & 1 & \cdots & 0 & 0 & \cdots & \cdots & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & \cdots & \cdots & 0 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & & \ddots & & \cdots & \cdots & & \vdots & & \vdots & \\
0 & 0 & \cdots & 0 & 1 & \cdots & \cdots & 0 & 0 & \cdots & 0 & 0 \\
* & * & \cdots & * & * & \cdots & \cdots & 0 & 0 & \cdots & 0 & 0 \\
 & \vdots & & & \vdots & \ddots & & & \vdots & & \vdots & \\
 & \vdots & & & \vdots & & \ddots & & \vdots & & \vdots & \\
0 & 0 & \cdots & 0 & 0 & \cdots & \cdots & 0 & 1 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & \cdots & \cdots & 0 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & & \vdots & & \cdots & \cdots & & \vdots & & \ddots & \\
0 & 0 & \cdots & 0 & 0 & \cdots & \cdots & 0 & 0 & \cdots & 0 & 1 \\
* & * & \cdots & * & * & \cdots & \cdots & * & * & \cdots & * & * \\
* & * & \cdots & * & * & \cdots & \cdots & * & * & \cdots & * & * \\
 & \vdots & & & \vdots & & \vdots & & \vdots & & \vdots & \\
* & * & \cdots & * & * & \cdots & \cdots & * & * & \cdots & * & *
\end{bmatrix}
\begin{bmatrix}
\xi_{1_1} \\ \xi_{1_2} \\ \vdots \\ \xi_{1_{r_1-2}} \\ \xi_{1_{r_1-1}} \\ \vdots \\ \vdots \\ \xi_{p_1} \\ \xi_{p_2} \\ \vdots \\ \xi_{p_{r_p-2}} \\ \xi_{p_{r_p-1}} \\ \xi_{r_p+1} \\ \vdots \\ \xi_n
\end{bmatrix}
+
\begin{bmatrix}
0 & \cdots & 0 \\
0 & \cdots & 0 \\
\vdots & & \vdots \\
0 & \cdots & 0 \\
* & \cdots & * \\
 & \vdots & \\
 & \vdots & \\
0 & \cdots & 0 \\
0 & \cdots & 0 \\
\vdots & & \vdots \\
0 & \cdots & 0 \\
* & \cdots & * \\
* & \cdots & * \\
\vdots & & \vdots \\
* & \cdots & *
\end{bmatrix}
\begin{bmatrix}
w_1 \\ w_2 \\ \vdots \\ w_{m-1} \\ w_m
\end{bmatrix}
$$

Observers for LTI strongly observable systems with unknown inputs

Observer in [Fridman et al. 2007]

- ✓ The states are estimated exactly after finite time;
- ✓ The cascade structure of observer allows to use a pre-filter.

Strong detectability

Question

If the system is not strongly observable, but strongly detectable, can we design an observer?

Strong detectability

Question

If the system is not strongly observable, but strongly detectable, can we design an observer?

Relations for strong detectability

i) The system is strongly detectable;

ii) the triplet (A, C, D) is minimum phase, i.e. the invariant zeros of the triplet (A, C, D) satisfy $Re\, s < 0$;

iii) all the trajectories belonging to $\mathcal{V}^*$ converges to zero asymptotically.

Observers for strongly detectable LTI systems

A canonical form.

$$\dot{x} = \begin{bmatrix} A_{11} & D_1 A_{12} & D_1 A_{13} \\ A_{21} C_1 & A_{22} & 0 \\ A_{31} C_1 & A_{32} C_2 & A_{33} \end{bmatrix} x + \begin{bmatrix} D_1 \\ 0 \\ 0 \end{bmatrix} w,$$

$$y = \begin{bmatrix} C_1 & 0 & 0 \\ 0 & C_2 & 0 \end{bmatrix} x,$$

1. the triplet (A_{11}, C_1, D_1) is strongly observable;
2. the pair (A_{22}, C_2) is observable;
3. A_{33} is stable (i.e. the invariant zeros of the system).

Observers for strongly detectable LTI systems

An observer [Fridman et al. 2011]

- x_1 can be recovered using y_1 and its derivatives (strong observability);
- x_2 is estimated asymptotically using Luenberger observer because it is not contaminaited

$$\dot{\hat{x}}_2 = A_{21}y_1 + A_{22}\hat{x}_2 + L_2C_2(x_2 - \hat{x}_2);$$

- for x_3 a copy of the system without injection (A_{33} is stable)

$$\dot{\hat{x}}_3 = A_{31}y_1 + A_{32}y_2 + A_{33}\hat{x}_3.$$

Remarks.

- it turns out that every strongly detectable (linear) system can be written in the previous form [Moreno:01];
- strong detectability [Hautus:83] is equivalent to the asymptotic distinguishability of the state trajectory from the output [Moreno:01]

Question

What happens when the system is not strongly observable. Is still possible to do something?

Outline

1. Conventional Sliding Mode Observers
2. A Simple Sliding Mode Observer
3. Higher Order Sliding Mode Observers
4. Strong Observability - Invariant Zeros - Relative Degree
   - Relation of concepts for SUISO Systems
   - HOSM Observers for LTI systems with unknown inputs
   - Functional HOSM Observers for linear systems
   - Nonlinear case
   - Unknown input identification
5. Output-based stabilization of disturbed systems
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   - Output-feedback finite-time stabilization of disturbed LTI systems.
6. Fault detection
   - Fault detection using multi-model approach

Functional Unknown Input Observers

Problem formulation

Given a linear system Σ, estimate a linear combination $z = Ex$ using only output measurements.

Motivation

- in many output based control problems, it is not necessary to estimate the whole state but a linear combination Kx;
- this is particularly true for output based sliding-mode control: only the the surface is required;
- despite a UIO does not exist (i.e. the system is not strongly detectable), the required functional UIO may exist.

THEOREM. [Sanuti and Saberi: 87]

Every linear system, with $p \geq m$, can be written as

$$\dot{x} = \begin{bmatrix} A_{11} & D_1 A_{12} & D_1 A_{13} & D_1 A_{14} \\ A_{21} C_1 & A_{22} & 0 & 0 \\ A_{31} C_1 & A_{32} C_2 & A_{33} & 0 \\ A_{41} C_1 & A_{42} C_2 & 0 & A_{44} \end{bmatrix} x + \begin{bmatrix} D_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} w,$$

$$y = \begin{bmatrix} C_1 & 0 & 0 & 0 \\ 0 & C_2 & 0 & 0 \end{bmatrix} x, \quad z = \begin{bmatrix} E_1 & E_2 & E_3 & E_4 \end{bmatrix} x,$$

and the following properties hold:

1. (A_{11}, C_1, D_1) is strongly observable and (A_{22}, C_2) is observable;
2. A_{33} are the stable invariant zeros and A_{44} the unstable ones.

Remark. [Angulo et.al: JFI14]

A functional UIO exists iff $E_4 = 0$.

Outline

1. Conventional Sliding Mode Observers
2. A Simple Sliding Mode Observer
3. Higher Order Sliding Mode Observers
4. Strong Observability - Invariant Zeros - Relative Degree
   - Relation of concepts for SUISO Systems
   - HOSM Observers for LTI systems with unknown inputs
   - Functional HOSM Observers for linear systems
   - Nonlinear case
   - Unknown input identification
5. Output-based stabilization of disturbed systems
   - Estimation of the disturbance.
   - Output-feedback finite-time stabilization of disturbed LTI systems.
6. Fault detection
   - Fault detection using multi-model approach

Observer for nonlinear system with unknown inputs

MIMO locally stable system [Fridman et al.: 2008].

Consider

$$\begin{aligned} \dot{x} &= f(x) + g(x)\varphi(t), \quad x(0) = x_0, \\ y &= h(x), \end{aligned} \tag{16}$$

where

- $x(t) \in \mathbb{R}^n$ is the state, $\varphi(t) \in \mathbb{R}^m$ is the disturbance;
- $y(t) \in \mathbb{R}^m$ is the measured output.

Formulation of the problem:

Estimate of $x(t)$ and $\varphi(t)$ based on output measurements only $y(t)$.

Assumption

The system is locally weakly observable
The system is BIBS

Observer for nonlinear system with unknown inputs

Coordinate transformation.

New coordinates

$$\xi^i = \begin{pmatrix} \xi_1^i \\ \xi_2^i \\ \vdots \\ \xi_{r_i}^i \end{pmatrix} = \begin{pmatrix} h_i(x) \\ L_f h_i(x) \\ \vdots \\ L_f^{r_i-1} h_i(x) \end{pmatrix}, \quad i = 1, ..., m; \; r = r_1 + ... + r_m$$

$$\xi = \begin{pmatrix} \xi^1 \\ \vdots \\ \xi^m \end{pmatrix}; \; \eta = \begin{pmatrix} \eta_1 \\ \vdots \\ \eta_{n-r} \end{pmatrix}$$

Local diffeomorphism.

There exist a local diffeomorphism such that

$$x = \Phi^{-1}(\xi, \eta)$$

Observer for nonlinear system with unknown inputs

Transformed system.

New coordinates

$$\begin{aligned} \dot{\xi}^i &= \Lambda_i \xi^i + \psi^i(\xi, \eta) + \lambda^i(\xi, \eta, \varphi(x)) \\ \eta &= q(\xi, \eta) \end{aligned}$$

where

$$\Lambda_i = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad \Phi_i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ L_f^{r_i} h_i(x) \end{bmatrix}$$

$$\lambda_i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \sum_{j=1}^{m} L_{g_i} L f^{r_i - 1} h_i(x) \varphi(x)_j \end{bmatrix}, \quad \forall\, i = 1, ..., m$$

Observer for nonlinear system with unknown inputs

State estimation [Fridman et al.: 2008].

HOSM Differentiator

$$
\begin{aligned}
\dot{v}_0^i &= w_0^i = -\alpha_{r_i} N^{1/r_i} |v_0^i - y_i|^{(r_i-1)/r_i} sign(v_0^i - y_i) + v_1^i \\
\dot{v}_1^i &= w_1^i = -\alpha_{(r_i-1)} N^{1/(r_i-1)} |v_1^i - w_0^i|^{(r_i-2)/(r_i-1)} sign(v_1^i - w_0^i) + v_2^i \\
&\vdots \\
\dot{v}_{r_i-1}^i &= w_{r_i-1}^i = -\alpha_2 N^{1/2} |v_{r_i-1}^i - w_{r_i-2}^i|^{1/2} sign(v_{r_i-1}^i - w_{r_i-2}^i) + v_{r_i}^i \\
\dot{v}_{r_i}^i &= -\alpha_1 N sign(v_{r_i}^i - w_{r_i-1})
\end{aligned}
\tag{17}
$$

Estimation by construction

$$
\begin{aligned}
\hat{\xi}_1^1 &= v_0^1, \quad \dots \quad \hat{\xi}_{r_1}^1 = v_{r_1-1}^1, \quad \dot{\hat{\xi}}_{r_1}^1 = v_{r_1}^1, \\
&\vdots \\
\hat{\xi}_1^m &= v_0^m, \quad \dots \quad \hat{\xi}_{r_1}^m = v_{r_1-1}^m, \quad \dot{\hat{\xi}}_{r_1}^m = v_{r_1}^m,
\end{aligned}
\tag{18}
$$

$$
\dot{\hat{\eta}} = q(\hat{\xi}, \hat{\eta})
$$

Observers for systems with unknown inputs

The system

Consider

$$\Sigma : \begin{cases} \dot{x} = f(x) + g(x)w, \quad x(0) = x_0, \\ y = h(x), \end{cases} \tag{19}$$

where

- $x(t) \in \mathbb{R}^n$ is the state, $w(t) \in \mathbb{R}^m$ is the unknown input;
- $y(t) \in \mathbb{R}^p$ is the measured output.

Formulation of the problem:

Estimate $x(t)$ based on output measurements only $\{y(t), t \in [0, T]\}$.

Remark.

When the relative degree of y is higher than one, output differentiations are necessary [Hautus: 83].

Observers for systems with unknown inputs

DEFINITION: Strong Observability. [Angulo et. al.: Automatica 13 (8)]

Σ is strongly observable if there exists a function F and integer k such that

$$x = F(y, \dot{y}, \cdots, y^{(k)}).$$

- equivalent to the distinguishability of the state trajectory using only the output when f, g and h are meromorphic [Angulo et.al: 10];
- also equivalent when Σ is linear [Hautus: 83]. In such a case, F is linear and therefore

$$x = \frac{d^k}{dt^k} M \begin{bmatrix} y \\ \int_0^t y(s)ds \\ \vdots \end{bmatrix}, \quad M \text{ is a matrix.} \tag{20}$$

- function F or matrix M can be computed using an algorithm [Angulo et. al: 10, Bejarano et.al.: 11, Davila et.al:10].

Further remarks.

- since the HOSM differentiators will estimate x, each individual gain L_j should satisfy
$$L_j > |\dot{x}_j| = |f_j(x) + g_j(x)w|$$
- for general nonlinear systems, the use of the HOSM differentiator is thus restricted to stable systems with bounded unknown inputs;
- for linear systems, the requirement of stability of the system can be circumvented using a Luenberger observer first [Bejarano et.al.: 10];
- analogously, an internal model for the unknown input can also be used to relax the requirement of bounded unknown inputs [Bejarano et.al: 10].

Outline

1. Conventional Sliding Mode Observers
2. A Simple Sliding Mode Observer
3. Higher Order Sliding Mode Observers
4. Strong Observability - Invariant Zeros - Relative Degree
   - Relation of concepts for SUISO Systems
   - HOSM Observers for LTI systems with unknown inputs
   - Functional HOSM Observers for linear systems
   - Nonlinear case
   - Unknown input identification
5. Output-based stabilization of disturbed systems
   - Estimation of the disturbance.
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6. Fault detection
   - Fault detection using multi-model approach

Unknown input identification

Problem formulation.

Given

$$\Sigma : \begin{cases} \dot{x} = f(x) + g(x)w, & x(0) = x_0, \\ y = h(x), \end{cases} \tag{21}$$

estimate the input $w(t)$ using only the measured output.

Motivation.

- when w is used to represent a disturbance, its estimate can be used for robust control [Ferreira et.al. CST: 11];
- when w is parametric uncertainty, its estimate can be used to estimate the unknown parameters [Davila et.al.:10];
- finite-time parameter estimation using Super-Twisting [Guzman et.al.: CTA TIE 2015, Gustavo Rueda 2016,2017.

Unknown input identification

DEFINITION: static left-inverse.

Σ has a static left-inverse if there exists a function G and integer k such that

$$w = G(y, \dot{y}, \cdots, y^{(k)}).$$

Remarks.

- if Σ is strongly observable and $\operatorname{rank} g = m$ then Σ has a static left-inverse [Angulo et.al: Automatica,13(8)];
- however, neither strong observability nor strong detectability are necessary for Σ to have a static left-inverse (see, e.g., [Bejarano et. al.: SIAM 09]).

Unknown input identification

Recall the canonical form

$$\dot{x} = \begin{bmatrix} A_{11} & D_1 A_{12} & D_1 A_{13} & D_1 A_{14} \\ A_{21} C_1 & A_{22} & 0 & 0 \\ A_{31} C_1 & A_{32} C_2 & A_{33} & 0 \\ A_{41} C_1 & A_{42} C_2 & 0 & A_{44} \end{bmatrix} x + \begin{bmatrix} D_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} w,$$

$$y = \begin{bmatrix} C_1 & 0 & 0 & 0 \\ 0 & C_2 & 0 & 0 \end{bmatrix} x,$$

THEOREM. [Bejarano et. al: 09]

- obviously w can be estimated iff x_1 can be estimated and $\operatorname{rank} D_1 = m$;
- it is not necessary strong observability nor strong detectability;
- the system has a static left-inverse if and only if the invariant zeros that do not belong to the set of unobservable eigenvalues are stable.

Outline

1. Conventional Sliding Mode Observers
2. A Simple Sliding Mode Observer
3. Higher Order Sliding Mode Observers
4. Strong Observability - Invariant Zeros - Relative Degree
   - Relation of concepts for SUISO Systems
   - HOSM Observers for LTI systems with unknown inputs
   - Functional HOSM Observers for linear systems
   - Nonlinear case
   - Unknown input identification
5. Output-based stabilization of disturbed systems
   - Estimation of the disturbance.
   - Output-feedback finite-time stabilization of disturbed LTI systems.
6. Fault detection
   - Fault detection using multi-model approach

Outline

1. Conventional Sliding Mode Observers
2. A Simple Sliding Mode Observer
3. Higher Order Sliding Mode Observers
4. Strong Observability - Invariant Zeros - Relative Degree
   - Relation of concepts for SUISO Systems
   - HOSM Observers for LTI systems with unknown inputs
   - Functional HOSM Observers for linear systems
   - Nonlinear case
   - Unknown input identification
5. Output-based stabilization of disturbed systems
   - Estimation of the disturbance.
   - Output-feedback finite-time stabilization of disturbed LTI systems.
6. Fault detection
   - Fault detection using multi-model approach

Output-feedback stabilization of disturbed systems

$$\dot{x} = Ax + B[u + w], \quad y = Cx.$$

First approach. [Ferreira et. al.: 10]

Use HOSM observers to estimate in finite-time:

- the unknown input w, and then remove it using control

$$u = -\hat{w} + v;$$

- the state x to construct the nominal control $v = K\hat{x}$.

Remarks: when to use or not SM in the control? [Ferreira et. al.: 10]

- identification error $\ll$ execution error: remove perturbation by identification;
- execution error $\ll$ identification error: use SM directly;

Outline

1. Conventional Sliding Mode Observers
2. A Simple Sliding Mode Observer
3. Higher Order Sliding Mode Observers
4. Strong Observability - Invariant Zeros - Relative Degree
   - Relation of concepts for SUISO Systems
   - HOSM Observers for LTI systems with unknown inputs
   - Functional HOSM Observers for linear systems
   - Nonlinear case
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5. Output-based stabilization of disturbed systems
   - Estimation of the disturbance.
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6. Fault detection
   - Fault detection using multi-model approach

$$\dot{x} = Ax + B[u + w], \quad y = Cx, \quad \|w(t)\| \leq W_1\|x(t)\| + W_2.$$

Second approach. [Angulo et.al.: 11]

- under strong observability, use a HOSM observer to estimate x in finite-time;
- by controllability there exists an output $\zeta(t) \in \mathbb{R}^m$ (not necessary the measured one) with vector relative degree;
- using ζ_i and its derivatives, write the system as m integrator chains;
- use (non-homogeneous) HOSM controllers to obtain (robust) finite-time state stability

$$v_i = -\alpha_i(k_{i,1}\|x\| + k_{i,2})H_{r_i}(\zeta_i, \dot{\zeta}_i, \cdots, \zeta_i^{(r_i-1)}), \quad i = 1, \ldots, m,$$

H_{r_i} is a r_i-th order SM algorithm, e.g.,

$$H_2(\zeta_i, \dot{\zeta}_i) := \frac{\dot{\zeta}_i + \beta|\zeta_i|^{\frac{1}{2}}\text{sign } \zeta_i}{|\dot{\zeta}_i| + \beta|\zeta_i|^{\frac{1}{2}}}.$$

Output-feedback stabilization of disturbed systems

Moreover

- the previous requirement of a measured output with vector relative degree is not necessary;
- robust finite-time stabilization of the whole state: useful for switching systems [Angulo and Levant: IJSS11];
- the same idea can be used for strongly observable nonlinear systems that are flat (not necessarily w.r.t. the measured output) [Angulo et. al.: AUTOMATICA,2013,NO.9];
- adapt the gain of the differentiator and control: reduce chattering;
- Separation Principle: (robust) on-line detection of the convergence of the differentiators;
- until now, this was done by waiting enough time.

Outline

1. Conventional Sliding Mode Observers
2. A Simple Sliding Mode Observer
3. Higher Order Sliding Mode Observers
4. Strong Observability - Invariant Zeros - Relative Degree
   - Relation of concepts for SUISO Systems
   - HOSM Observers for LTI systems with unknown inputs
   - Functional HOSM Observers for linear systems
   - Nonlinear case
   - Unknown input identification
5. Output-based stabilization of disturbed systems
   - Estimation of the disturbance.
   - Output-feedback finite-time stabilization of disturbed LTI systems.
6. Fault detection
   - Fault detection using multi-model approach

Outline

1. Conventional Sliding Mode Observers
2. A Simple Sliding Mode Observer
3. Higher Order Sliding Mode Observers
4. Strong Observability - Invariant Zeros - Relative Degree
   - Relation of concepts for SUISO Systems
   - HOSM Observers for LTI systems with unknown inputs
   - Functional HOSM Observers for linear systems
   - Nonlinear case
   - Unknown input identification
5. Output-based stabilization of disturbed systems
   - Estimation of the disturbance.
   - Output-feedback finite-time stabilization of disturbed LTI systems.
6. Fault detection
   - Fault detection using multi-model approach

Fault detection using multi-model approach

The system with fault

$$\begin{aligned} \dot{x} &= f(x) + Bu + F_i(x,u), \quad i = 1, \ldots, q \\ y &= h(x) \end{aligned} \tag{22}$$

where

- $x(t) \in \mathbb{R}^n$ is the continuous state;
- $u(t) \in \mathbb{R}^m$ is a known input;
- $y(t) \in \mathbb{R}^p$ is the measured output;
- Function and matrix (f, B) are known;
- $F_i(x,u)$ are faults (known a-priori) defined by:

$$F_i(x,u) \in \mathcal{F}, \quad \textit{where} \quad \mathcal{F} = \{F_1(x,u), \ldots, F_q(x,u)\}$$

i.e. q faulty cases that change the system properties (*plant faults*) or/and the dynamical input properties of the system (*actuator faults*).

Fault detection using multi-model approach

Problem statement

Given $\{y(s), s \in [0, t]\}$ identify and isolate the q possible faults (FDI).

Main idea: use q HOSM Observers as multi-models and the equivalent injection to carry out FDI.

Proposed Solution (H. Ríos et al. IJRNC 2014)

- HOSM differentiator applied to the estimation error dynamics;
- Finite-time reconstruction of continuous state;
- Fault detection and isolation using Equivalent Injection;

HOSM Observers

- provide theoretically exact observation and unknown inputs and fault estimation under sufficient and necessary conditions of the strong observability/ detectability of states or unknown inputs or faults;

HOSM Observers

- provide theoretically exact observation and unknown inputs and fault estimation under sufficient and necessary conditions of the strong observability/ detectability of states or unknown inputs or faults;
- provide best possible approximation w.r.t. discretization step and/or bounded deterministic noises:

HOSM Observers

- provide theoretically exact observation and unknown inputs and fault estimation under sufficient and necessary conditions of the strong observability/ detectability of states or unknown inputs or faults;
- provide best possible approximation w.r.t. discretization step and/or bounded deterministic noises:
- can ensure prescribed time convergence independent from any initial conditions.

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